Spring 2010 Math 245-2 Exam 2

Please read the following directions:

Please write legibly, with plenty of white space. Please put your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use your book and/or notes, but no calculators or other aids. This exam will last 60 minutes; pace yourself accordingly. If you are done early, you may leave – but NOT during the last five minutes of the exam, during which you are asked to remain quiet and in your seat. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Total:	50		100

Problem 1. Carefully define each of the following terms:

    a. constructive proof

    b. floor

    c. odd

    d. irreducible

Problem 2. Prove that $\sqrt{5}$ is irrational.

Problem 3. Use the Euclidean algorithm to first find $\gcd(21, 15)$, then to express $\gcd(21, 15)$ as a linear combination of 15 and 21.

Problem 4. Prove that, in the reals, the product of an irrational and a nonzero rational is irrational.

Problem 5. Prove or disprove that, $\forall x \in \mathbb{R}$, $\lfloor -x \rfloor = -\lfloor x \rfloor$.

Problem 6. Prove or disprove that, $\forall x \in \mathbb{R}$, $\lfloor -x \rfloor = -\lceil x \rceil$.

Problem 7. Prove or disprove that, $\forall x \in \mathbb{R}$, $-|x| \le x \le |x|$.

Problem 8. Consider the sequence given by $a_1 = 1, a_{n+1} = 3a_n + 3^n$ (for $n \ge 1$). Prove that $a_n = n3^{n-1}$.

Problem 9. Prove that $\left(\begin{smallmatrix} 3 & 1 \\ 0 & 3 \end{smallmatrix}\right)^n = \left(\begin{smallmatrix} 3^n & n3^{n-1} \\ 0 & 3^n \end{smallmatrix}\right)$.

Problem 10. For an arbitrary set of numbers S, recall that x is a *unit* if there is some y such that $xy = 1$. Prove that the product of two units is a unit.
Technical note: S is closed under multiplication, multiplication is associative